Radical Expressions and Radical Functions

Warm-Up

Identify each transformation of the parent function $f(x) = x^2$.

1. $f(x) = x^2 + 5$

2. $f(x) = (x+5)^2$

3. $f(x) = 5x^2$

4. $f(x) = -5x^2$

5. $f(x) = (5x)^2$

6. $f(x) = \left(\frac{1}{5}x\right)^2$

Square-root Functions

The square root of a number, $\sqrt{x}$ is a number that when multiplied by itself produces the given number, x. Since the domain of a function is the set of all real-number values of x for which a function, f, is defined, the domain of the square-root function, $f(x)=\sqrt{x}$ does not include negative numbers.

Find the domain of each function below.

$h(x) = \sqrt{-4x+7}$

$-4x+7 \geq 0$

$-4x \geq -7$

$x \leq 7/4$

$f(x) = \sqrt{2x-5}$

$2x-5 \geq 0$

$2x \geq 5$

$x \geq \frac{5}{2}$

$g(x) = \sqrt{5x+18}$

$5x+18 \geq 0$

$5x \geq -18$

$x \geq \frac{-18}{5}$

Transformations for the square-root parent function, $y = \sqrt{x}.$

$$y = a\sqrt{b(x-h)} + k$$

a = the vertical stretch or compression by a factor of |a|; for a < 0, the graph is reflected across the x-axis.

b = the horizontal stretch or compression by a factor of $\left|\frac{1}{b};\right|$ for b<0, the graph is a reflection across the y-axis.

h = the horizontal translation h units to the right for h > 0 and |h| units to the left for h < 0.

k = the vertical translation k units up for k > 0 and |k| units down for k < 0.

For each function, describe the transformations applied to $y = \sqrt{x}$.

$y = -2\sqrt{x+1} + 4$

Reflection over x

vertical stretch factor 2

horz translation left 1

vert trans up 4

$y = \sqrt{4x-3} - 1$

$y = \sqrt{4\left(x - \frac{3}{4}\right)} - 1$

horz comp fact 1/4

horz trans right $\frac{3}{4}$

vert trans down 1

$$y = 3\sqrt{x-2} - 5$$

vert stretch factor 3

horz trans right 2

vert trans down 5

$$\sqrt{-2(x - 1/2)}$$

$$y = -\sqrt{-2x+1} + 8$$

Reflect over x-axis

Reflect over y-axis

horz compress factor 1/2

horz trans right 1/2

vert trans up 8

$y = 3\sqrt{x-1} - 2$

$y = \sqrt{2x+1} + 3$

$\sqrt{2(x + \frac{1}{2})}$

horz compress $\frac{1}{2}$

$\frac{1}{2}$ inside horz trans left $\frac{1}{2}$ units

Recall that you can find the inverse of a function by interchanging x and y and then solving for y.

Find the inverse of $y = x^2 - 2x$. Then graph the function and its inverse together.

$x = y^2 - 2y$

$0 = y^2 - 2y - x$

a: 1 b: -2 c: -x

$y = \frac{2 \pm \sqrt{4 - 4(1)(-x)}}{2}$

$y = \frac{2 \pm \sqrt{4 + 4x}}{2}$

$y = \frac{2 \pm \sqrt{4(1+x)}}{2}$

$y = \frac{2 \pm 2\sqrt{1+x}}{2}$

$y = 1 \pm \sqrt{1+x}$

Find the inverse of $y = x^2 + 1$.

$$x = y^2 + 1$$

$$x = y^2 + 0y + 1$$

$$0 = y^2 + 0y + 1 - x$$

$$y = \frac{0 \pm \sqrt{0 - 4(1)(1-x)}}{2}$$

$$y = \frac{0 \pm \sqrt{-4 + 4x}}{2}$$

$$y = \frac{\pm\sqrt{4(-1+x)}}{2}$$

$$y = \pm\frac{2\sqrt{-1+x}}{2}$$

$$y = \pm\sqrt{x-1}$$

Find the inverse of $y = x^2 + 3x - 4$

Cube-Root Functions

The cube root of a number $\sqrt[3]{x}$,, is a number that when multiplied by itself 3 times produces the given number, x.

The domain of $f(x)=\sqrt[3]{x}$ is all real numbers, and the range of f is all real numbers. Graph $f(x)=\sqrt[3]{x}$ in a graphics calculator to see why.

Evaluate each expression.

$3\sqrt[3]{27}-5$

27^$(1\div3)$

$3(3)-5$

4

$2\left(\sqrt[3]{-64}\right)^2+7$

(-64)^$(1\div3)$

$2(-4)^2+7$

$2(16)+7$

$32+7$

39

$$-6\left(\sqrt[3]{-8}\right)-2$$

$-6(-2)-2$

$12-2$

10

$$4\left(\sqrt[3]{216}\right)^2$$

$4(6)^2$

$4(36)$

144

$-2\sqrt[3]{-125}-10$

$-2(-5)-10$

$10-10$

0

$6\left(\sqrt[3]{8}\right)^2+2$

26