

Exponential Functions

Warm-UP

Evaluate.

1. 3^4 81

2. 3^0 1

3. $(-4)^2$ 16

4. 4^{-2} $\frac{1}{16}$

Evaluate each expression for $x = 2$.

5. x^5 32

6. 5^x 25

7. 5^{-x} $\frac{1}{25}$

8. $\left(\frac{1}{5}\right)^x$ $\frac{1}{25}$

Exponential Function

The function $f(x) = b^x$ is an **exponential function** with **base** b , where b is a positive real number other than 1 and x is any real number.

Examine the table at the left and graph the function $y = 2^x$. Notice that as x -values decrease, the y -values for $y = 2^x$ get closer and closer to 0, approaching the x -axis as an asymptote. An asymptote is a line that a graph approaches (but does not reach) as its x - or y -values become very large or very small.

The graphs of $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$

exhibit the two typical behaviors for exponential functions. Graph both in your calculator.

Exponential Growth and Decay

When $b > 1$, the function $f(x) = b^x$ represents exponential growth.

When $0 < b < 1$, the function $f(x) = b^x$ represents exponential decay.

Exponential growth functions and exponential decay functions of the form $y = bx$ have the same domain, range, and y-intercept. For example:

Func tion	Domain	Range	y-intercept
$f(x) = 2^x$	All real #s	All + real #s	1
$g(x) = \left(\frac{1}{2}\right)^x$	All real #s	All + real #s	1

Graph $f(x) = 2^x$ along with each function below. Tell whether each function represents exponential growth or exponential decay. Then give the y-intercept.

1. $y = 3 * f(x)$

$$y_1 = 3 * 2^x$$

growth
y-int = 3

2. $y = 5 * f(-x)$

$$5 * 2^{-x}$$

decay int = 5

3. $\frac{1}{2} * f(-x)$

$$\frac{1}{2} * 2^{-x}$$

decay int $\frac{1}{2}$

4. $4 * f(x)$

$$4 * 2^x$$

growth int 4

Try These:

$$y = \frac{1}{3} * f(x)$$

G or D

$$\text{int: } \frac{1}{3}$$

$$f(x) = 2^x$$

$$y = \frac{1}{4} * f(-x)$$

G or D

$$\text{int: } \frac{1}{4}$$

Compound Interest Formula

The total amount of an investment, A , earning compound interest is

$$A = P(1+r)^t$$

$$A(t) = P \left(1 + \frac{r}{n} \right)^{nt}$$

where P is the principal, r is the annual interest rate, n is the number of times interest is compounded per year, and t is the time in years.

Find the final amount of a \$100 investment after 10 years at 5% interest compounded annually, quarterly, and daily.

$$A(t) = P \left(1 + \frac{r}{n} \right)^{nt}$$

$= 100 \left(1 + \frac{0.05}{1} \right)^{(1 \times 10)} = 162.89$
 $= 100 \left(1 + \frac{0.05}{4} \right)^{(4 \times 10)} = 164.36$
 $= 100 \left(1 + \frac{0.05}{365} \right)^{(365 \times 10)} = 164.87$

Find the final amount of a \$500 investment after 8 years at 7% interest compounded annually, quarterly, monthly, and daily.

$$A = 500 \left(1 + \frac{0.07}{4} \right)^{(4 \times 8)}$$

$= 859.09$ 871.1 873.91 875.29

To tell if an equation is linear, quadratic or exponential,

Ask yourself:

Is the power on x 1?	linear
Is the x squared?	quadratic
Is the x the exponent?	exponential